

Probability Summary

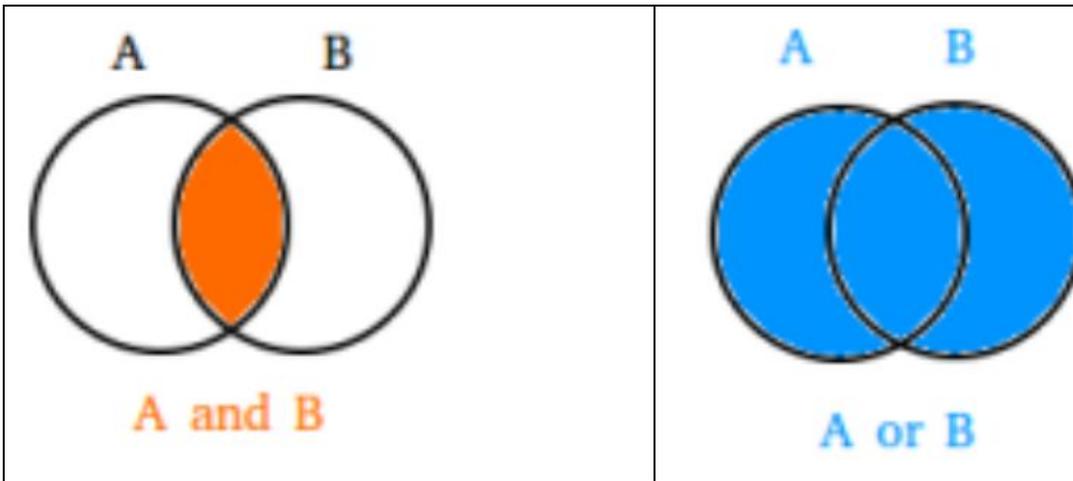
Probability Day 1

$$\text{Probability} = \frac{\text{number of outcomes in event}}{\text{total number of outcomes}}$$

Complementary Events: If \bar{A} is the complement of A (or not A) then $P(\bar{A}) = 1 - P(A)$.

AND - the probability of event A and B is the probability of both events occurring.

OR - the probability of event A or B is the probability of EITHER A occurring, or B occurring, or both occurring.



AND is more restrictive. It's harder to meet both criteria, so the probability of A and B will be lower.

OR is less restrictive. It is easier to meet either criteria, so the probability of A or B will be higher than A and B.

Probability Day 2

Independent

Events A & B are independent if the probability of event B occurring is the same whether or not event A occurs.

AND formulas

If Events A & B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$

Mutually Exclusive Events:

Two events are mutually exclusive if they do not overlap. Another way to say this is that events A & B cannot both happen at the same time.

If A and B are mutually exclusive, then $P(A \text{ and } B) = 0$

OR formula - This formula is true in all cases (not just when A & B are independent).

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Probability Day 3

Conditional Probability

The probability of A given B is the probability that event A occurs given that we are only considering outcomes in B. So the denominator of the fraction is the number of outcomes in event B, and the numerator is the number of outcomes in B that are also in A. (This makes more sense in context.)

The more general version of the **AND formula** (which is true whether or not A & B are independent) is $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Contingency Table or Two-Way Table

I have used these tables throughout the unit and then asked probability questions from them. A contingency table is a way to organize people two different ways.

Probability Day 4

Basic Counting Rule

If we are asked to choose one item from each of two separate categories where there are m items in the first category and n items in the second category, then the total number of available choices is $m \cdot n$.

Another way: If there are m ways of doing one thing, and n ways of doing another, then there are $m \cdot n$ ways of doing the first thing followed by the second thing.

Repetition allowed

If a license plate has 2 letters followed by 4 numbers, then there are $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ ways to create a license plate, or 6,760,000 possible different license plates.

Repetition not allowed

If a license plate has 2 letters followed by 4 numbers, and repetition is not allowed, then there are $26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7$ ways to create a license plate, or 3,276,000 possible different license plates.

Choosing within ONE category >> without replacement/repetition

Permutations

A permutation is a way to choose r objects from n choices (without replacement), when the order of the objects chosen matters.

If I choose 3 students from the 16 who are present today, and order matters, then there are $16P3$ ways to do this. $16P3 = nPr(16,3)$ in desmos = 3360 different ways to do this.

Combinations

A way to select r objects from n choices (without replacement), when order doesn't matter.

If I choose 3 students from the 16 who are present today, and order does not matter, then there are $16C3$ ways to do this. $16C3 = nCr(16,3)$ in desmos = 560 different ways to do this.

Probability Day 5

Expected value is the average gain or loss of an event in the long run (over many trials).

We can compute the expected value by multiplying each outcome by the probability of that outcome, and then adding up the products.

You purchase a raffle ticket to help out a charity. The raffle ticket costs \$5. The charity is selling 2000 tickets. One of them will be drawn and the person holding the ticket will win \$4000. Compute the expected value for this raffle.

Event	Probability of Event	Amount Won or Lost	Product
you win	$1/2000$	$-\$5 + \$4000 = \$3995$	$\$1.9975 \approx \2
you lose	$1999/2000$	$-\$5$	$-\$4.9975 \approx -\5
			$-\$3$

The expected outcome for this raffle is that each person is going to lose \$3 on average.